HYPER VERSION

Received: *May 10, 2001,* Accepted: *June 19, 2001*

Stable solitons in field theory models for tachyon condensation

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Abstract: We study soliton solutions in scalar field theory with a variety of unbounded potentials. A subset of these potentials have Gaussian lump solutions and their fluctuation spectrum is governed by the harmonic oscillator problem. These lumps are unstable with one tachyonic mode. Soliton solutions in several other classes of potentials are stable and are of kink type. The problem of the stability of these solutions is related to a supersymmetric quantum mechanics problem. The fluctuation spectrum is not equispaced and does not contain any tachyonic mode. The lowest energy mode is the massless Goldstone mode which restores broken translation invariance.

Keywords: Solitons Monopoles and Instantons, Bosonic Strings.

JHEP06(2001)039

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Contents

1. Introduction

Tachyon condensation in string theory has attracted considerable attention following the conjectures of Sen [1]–[4]. These conjectures have been studied using string field theory both in the background independent formulation [5]–[9] as well as in the cubic string field theory [10] using the level truncation scheme [11]–[14]. Strong evidence in support of these conjectures is now available [15]–[20] due to these results.

Toy models of tachyon condensation are extremely useful in learning more about the dynamics and stability of non-BPS D-branes. In particular, an exactly solvable toy model, which preserves the basic features of string field theory, may educate us about the disappearance of the unstable D-brane and structure of the closed string vacuum. To this end, Zwiebach [21] and then Minahan and Zwiebach [22] have studied some solvable toy models for tachyon condensation in field theory. While the string field contains an infinite number fields, which include tachyon, gauge field and higher massive stringy excitations, their toy model contains only a single scalar field, representing the tachyon. In the cubic string field theory, the product of string fields is defined in terms of the $*$-product which involves an infinite number of derivatives of the field. In their field theory model, the Lagrangian density contains at most two derivatives of the scalar field.

The model which was studied by Minahan and Zwiebach [22] contains a scalar field with a canonical kinetic term and a potential

$$V_{\infty}(\phi) = -\frac{1}{4}\phi^2 \ln(\phi^2)\,, \tag{1.1}$$

which is obtained by taking $p \to \infty$ limit of

$$V_p(\phi) = \frac{p}{4}\phi^2(1 - \phi^{2/p})\,. \tag{1.2}$$

The model studied by Zwiebach [21], a particular case of the potential (1.2), supports a lump solution, the fluctuation spectrum of which reveals an unstable (tachyonic) mode. For finite values of p, there are a finite number of discrete fluctuation modes and their spectrum is not equispaced. However, in the $p \to \infty$ limit, the lump solution is a Gaussian with the fluctuation spectrum governed by the Schrödinger equation for harmonic oscillator. Consequently, the fluctuation spectrum is equispaced, which is a feature of the spectrum of string theory. The potential (1.1) is identical to the tachyon potential in the boundary string field theory. This formulation is formally background independent. In this case the tachyon mode has mass $m^2 = -1$ which is another feature common with the string theory tachyon.

In this paper, we enquire if this behaviour of the lump solution and its fluctuation spectrum are unique to the potential (1.1) above or whether there is some universality in it. In other words, are there other potentials with some parameter p, which in the limit $p \to \infty$, have a Gaussian lump solution? Do we recover a lump solution whose fluctuation spectrum is equispaced? We will see that this is not so. We find that a large class of models do not have unstable lump solutions. Instead, they have stable kink solutions, despite having unbounded potentials. The lowest energy mode of fluctuations about these kink solutions is massless. This is the Goldstone mode of the kink corresponding to broken translation invariance.

In section 2, we will briefly review the work of Minahan and Zwiebach [22]. In section 3, we study various classes of potentials, which include those with a gaussian lump solution as well as those with more complicated soliton profiles. Since we want to see if in the limiting case we recover the Schrödinger equation for harmonic oscillator, we will concentrate on the potentials obtained by taking the $p \to \infty$ limit.

2. Review of the Minahan-Zwiebach potential

In this section we will briefly review the model of Minahan and Zwiebach [22]. They consider a toy model in field theory to study tachyon condensation. Unlike open string field theory which contains infinite number of fields, this model contains only one scalar field. The Lagrangian contains terms up to two derivatives. In other words, it contains a canonical kinetic term and a potential term which is a function purely of the scalar field and not of its derivatives.

The lagrangian density is

$$\mathcal{L} = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V_p(\phi)\,, \tag{2.1}$$

where the potential $V_p(\phi)$ is

$$V_p(\phi) = Ap\phi^2(1 - \phi^{2/p})\,. \tag{2.2}$$

The parameter p is an integer and A is chosen to be $1/4$ but we will keep it as a free parameter. This potential is well defined for $p = 1, 2$ and for $p > 2$, $\phi^{2/p} \equiv (\phi^2)^{1/p}$, where the pth root is taken to be real and positive. The equation of motion for ϕ is

$$\phi''(x) = V_p{}'(\phi(x))\,, \tag{2.3}$$

where, the prime denotes derivative with respect to the argument. A classical solution to this equation of motion is

$$\bar{\phi}(x) = \mathrm{sech}^p\left(\frac{x}{\sqrt{2p}}\right)\,, \tag{2.4}$$

which is a lump solution. If the field ϕ is expanded about the lump $\bar{\phi}$, the fluctuation modes can be shown to satisfy the Schrödinger equation [23]

$$-\frac{d^2\psi(x)}{dx^2} + U(x)\psi(x) = m^2\psi(x)\,, \tag{2.5}$$

where $U(x) = V_p''(\bar{\phi}(x))$. Using the form of the lump solution in this equation we can determine the form of the quantum mechanical potential $U(x)$:

$$U(x) = \frac{1}{2p}\left(p^2 - (p+1)(p+2)\mathrm{sech}^2\left(\frac{\mathrm{x}}{\sqrt{2\mathrm{p}}}\right)\right)\,. \tag{2.6}$$

This problem belongs to a class of exactly solvable quantum mechanical models [24]. For integer p, this potential admits $p + 1$ bound states and rest is a continuum with reflectionless scattering matrix. Using the spectrum of this problem the mass spectrum of the fluctuations can be deduced —

$$m^2 = \frac{1}{2p}(p^2 - (p + 1 - n)^2), 0 \leq n < p + 1\,. \tag{2.7}$$

The lowest energy fluctuation mode is tachyonic with $m^2 = -1 - 1/2p$ and the next mode is massless. In the limit $p \to \infty$, the tachyon mass-squared becomes $m^2 = -1$, which mimics behaviour of the tachyon in the open string theory. The potential in the field theory model when $p \to \infty$ reduces to

$$V_\infty(\phi) = -A\phi^2\ln(\phi^2)\,. \tag{2.8}$$

This is a familiar potential in the background independent open string field theory.

The field theory problem with the potential V_∞ admits a lump solution which is a gaussian and the Schrödinger equation for the fluctuation modes is that of the harmonic oscillator. The spectrum is equispaced. The lowest mode is tachyonic with $m^2 = -1$ and next mode is massless.

As mentioned in the introduction, our motivation for studying generalizations of this problem is to enquire if the gaussian nature of the lump solution and consequently equispaced fluctuation spectrum, in $p \to \infty$ limit is a universal feature or not. In the rest of the paper we will consider a more general class of potentials $V_p(\phi)$, seek a solution to the classical equation of motion and study the spectrum of the fluctuation modes. We will concentrate specifically on the $p \to \infty$ limit of these potentials to see if the fluctuation spectrum is equispaced.

3. Generalized potentials

In this section we will consider more general potentials $V(\phi)$ and seek soliton solutions to the equations of motion. In the first subsection, we will address a simple issue related to the reflectionless potentials in the Schrödinger equation which appeared in the model studied by Zwiebach [21]. These quantum mechanical potentials are parametrized by an integer p,

$$U(x) = p^2 - p(p+1)\mathrm{sech}^2\mathrm{x}\,. \tag{3.1}$$

If p is not integer then the corresponding quantum mechanical problem does not have reflectionless scattering matrix but the problem is still exactly solvable. In the following we demonstrate that the reflectionlessness of the potential is not relevant in the $p \to \infty$ limit and we still end up with a harmonic oscillator problem for the fluctuation modes of the soliton solution.

In the second subsection 3.2, we will consider more general potentials which are all unbounded below. An interesting feature of all these potentials is that each has a point of inflection at $\phi = 1$. This leads to a novel type of soliton solution which has no unstable mode despite potential being unbounded.

3.1 Simple extension of the Minahan-Zwiebach model

We will start with a minimal generalization of the Minahan-Zwiebach model. Consider a potential of the form

$$V_{p,q}(\phi) = A\phi^2 p(1 - \phi^{q/p})\,, \tag{3.2}$$

where p and q are positive integers and A is some arbitrary constant. For any relatively prime integers p and q, equation of motion for this potential has a soliton solution

$$\bar{\phi}(x) \sim \mathrm{sech}^{2p/q}\left(\frac{qx}{\sqrt{2p}}\right)\,. \tag{3.3}$$

The stability analysis of this soliton solution gives us a Schrödinger equation with the potential

$$U(x) = \frac{1}{2p}\left(p^2 - (p+q)\left(p+\frac{q}{2}\right)\,\mathrm{sech}^2\left(\frac{qx}{\sqrt{2p}}\right)\right). \tag{3.4}$$

This potential is not reflectionless. However, the Schrödinger equation is still exactly solvable [24].

As mentioned in the previous section we will be interested in $p \to \infty$ limit. In this limit, scalar field theory potential becomes

$$\begin{aligned} V_{\infty,q}(\phi) &= \lim_{p\to\infty} V_{p,q}(\phi) \\ &= A\phi^2 \lim_{n\to 0} \frac{(1-\phi^{qn})}{n} \\ &= -A\phi^2 \ln(\phi^q) = -Aq\phi^2 \ln(\phi)\,. \end{aligned} \tag{3.5}$$

This potential is identical to that obtained by Minahan and Zwiebach except that A is replaced by $Aq/2$. This, however, does not affect any of their results, *viz.* lump with a gaussian profile and equispaced fluctuation spectrum, as A can easily soak up the additional factor. So the lesson we learn from this is that $p \to \infty$ can really be reached in a continuous manner. Reflectionless feature of the potential for the fluctuation modes is of no consequence.

3.2 New unbounded potentials

In this subsection we will consider more general potentials. Looking at the original proposal of Zwiebach [21], we see that at $p = \infty$ the potential develops a simple zero, and is of indeterminate form $\infty \times 0$. The $p \to \infty$ limit is then determined using L'Hôspital's rule. A straightforward generalization of this is to consider potentials which have a higher order zero as $p \to \infty$. To this end let us consider

$$V(\phi) = A\phi^2 p^n (1-\phi^{q/p})^n\,, \tag{3.6}$$

for an arbitrary odd integer $n > 1$. Recall that $n = 1$ corresponds to the original problem studied by Zwiebach, and Minahan and Zwiebach. As usual we will restrict ourselves to $p \to \infty$ limit. In this limit, the potential becomes

$$V(\phi) = Aq^n \phi^2 (-\ln\phi)^n\,. \tag{3.7}$$

Notice that for odd n, this potential is unbounded below. The generic form of $V(\phi)$ for odd n is shown in figure 1. So far we have seen that a classical solution to the equation of motion with a potential which is unbounded below has tachyonic mode. The potential (3.7) possesses a local minimum at $\phi = 0$ where the first

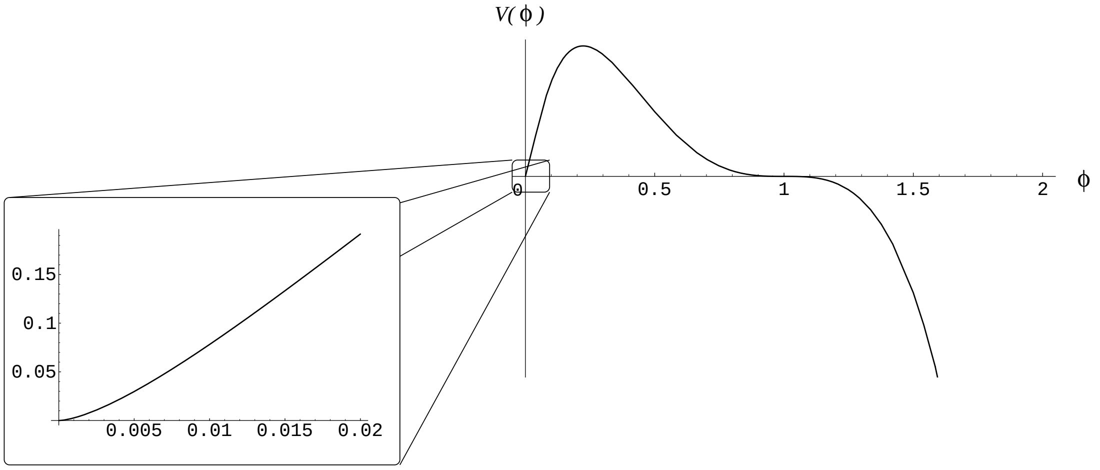


Figure 1: Generic form of $V(\phi)$ for odd n. The enlarged region shows the behaviour near the origin.

derivative vanishes. It has a maximum at $\phi = e^{-n/2}$. At this unstable point, $V'' = -2Aq^n(n/2)^{n-1}$, which is the mass-squared of the tachyon. There is also an inflection point at $\phi = 1$ at which the first $n-1$ derivatives of the potential vanish. Since the potential, for odd n, is unbounded below it is natural to expect that the lowest lying fluctuation will destabilize the classical solution. However, we have a surprise in store here. As we will see below, despite having an unbounded potential, the classical solution that we get is not a lump solution but is a kink. This is because of the fact that the potential (3.7) has a point of inflection at $\phi = 1$.

A solution to the equations of motion can be obtained by solving the integral equation

$$\int \frac{d\bar{\phi}}{\bar{\phi}(-\ln\bar{\phi})^{n/2}} = \pm\sqrt{2q^n A}x\,, \tag{3.8}$$

and is given by

$$\bar{\phi}_{\mp}(x) = \exp\left[\mp\alpha_n (x^2)^{-\frac{1}{n-2}}\right], \qquad \text{for}\, n \geq 3\,, \tag{3.9}$$

where

$$\alpha_n = \left[\left(\frac{n}{2}-1\right)^2 2Aq^n\right]^{-1/(n-2)}. \tag{3.10}$$

We will take $(n-2)$th root of x^2 to be real and positive. This is assured if n is an odd integer. Notice that there are two solutions corresponding to each of the $\mp$ signs in the exponent. Let us first examine the solution with the positive sign, i.e. $\bar{\phi}_+(x)$. This solution blows up at $x = 0$ and it approaches $\phi = 1$ as $x \to \infty$. This field configuration interpolates between the extremum at $\phi = \infty$ and the inflection point at $\phi = 1$. This solution is neither relevant nor accessible if we are studying tachyon

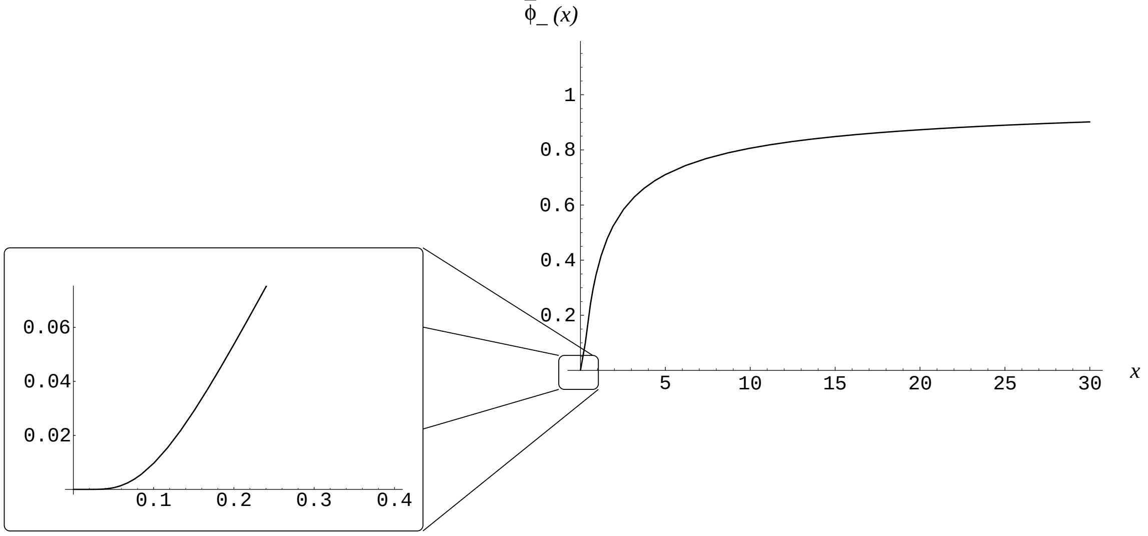


Figure 2: The kink $\bar{\phi}_-(x)$. The enlarged region shows the behaviour near the origin.

condensation, which corresponds to the field rolling down from the maximum of the potential. The solution $\bar{\phi}_+(x)$ does not ever reach the maximum of the potential. It always stays to the right of the inflection point, while the maximum is to the left of the inflection point.

The solution of interest to us is $\bar{\phi}_-(x)$, the profile of which is shown in figure 2. It approaches $\phi = 0$ as $x \to 0$, which is a local minimum of the potential and as $x \to \infty$ it approaches $\phi = 1$. For intermediate value of x, it rolls over the maximum of the potential. At this point we would like to make a few remarks about the solution $\bar{\phi}_-(x)$. As mentioned earlier, $\phi = 1$ is a point of inflection and for all values of $n > 2$, at least two derivatives of the potential with respect to ϕ vanish at that point. Using the equation of motion, this translates into the fact that at $\phi = 1$ at least three derivatives of the solution $\bar{\phi}_-(x)$ with respect to x vanish. Recall that the lagrangian of our model contains terms which have at most two derivatives in it. Thus, there are no higher derivative terms in the theory which can destabilise this solution. We believe the situation is quite different if we view this problem from the string field theory perspective. The latter contains $*$-products of fields, which involves an arbitrary number of derivatives of the string field. These terms may destabilise the solution $\bar{\phi}_-(x)$. At this point, one may wonder why we are not looking at the solution for negative values of x. The reason for this is that the kink solution is such that as we approach $x = 0$, $\bar{\phi} \to 0$ in such a way that all its derivatives vanish at $x = 0$, i.e. the soliton solution has an essential singularity at $x = 0$. In the mechanical model analogy [25], the particle will take infinite time to reach $x = 0$ from $x > 0$ and hence $x < 0$ region will not be reachable. We will come back to this point after we study the mass spectrum of the fluctuations around this kink solution.

The potential which governs the mass spectrum of the fluctuation modes is

$$U_n(x) = \frac{2}{(n-2)^2}\left[2\alpha_n^2\left(\frac{1}{x^2}\right)^{n/(n-2)} - 3n\alpha_n\left(\frac{1}{x^2}\right)^{(n-1)/(n-2)} + n(n-1)\frac{1}{x^2}\right]. \tag{3.11}$$

It is possible to recast the Schrödinger equation with this potential in terms of supersymmetric quantum mechanics [24]. The corresponding quantum mechanical superpotential is given by

$$W(x) = \frac{n}{n-2}\frac{1}{x} - \left[\frac{2^{n-1}}{(n-2)^n A q^n}\right]^{1/(n-2)} \frac{1}{x^{n/(n-2)}} \tag{3.12}$$

$$= \frac{1}{n-2}\left[\frac{n}{x} - \frac{2\alpha_n}{x^{n/(n-2)}}\right]. \tag{3.13}$$

This allows us to calculate the exact ground state wavefunction,

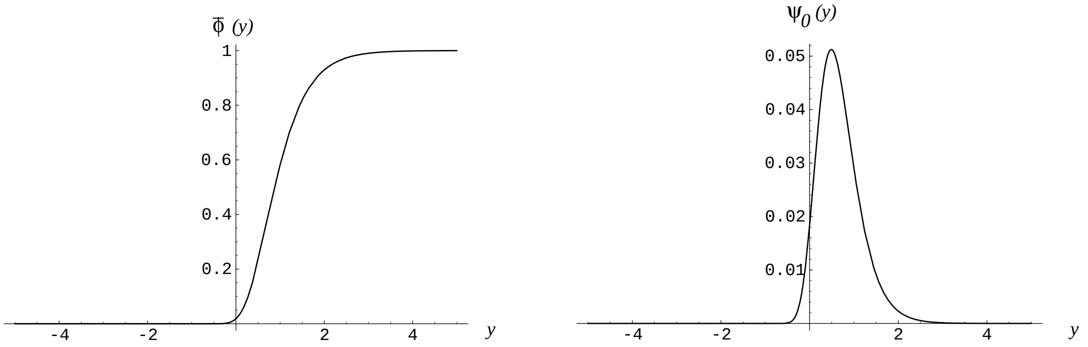
$$\psi_0(x) = \exp\left(-\int W(x)dx\right) \tag{3.14}$$

$$= x^{-n/(n-2)} \exp\left(-\alpha_n x^{-2/(n-2)}\right). \tag{3.15}$$

Exact supersymmetry of the Schrödinger equation for the fluctuation spectrum implies that the ground state energy is zero, i.e., the lowest lying fluctuation mode is massless. Thus, the fluctuation spectrum does not contain a tachyon, which means that the kink solution is stable.

Another way of seeing that the kink solution is stable is the following. The existence of a space-dependent solution spontaneously breaks translation invariance. Studying low energy excitations around that solution then gives the massless Goldstone mode, which is responsible for restoring broken translation invariance. If the solution is unstable, the lowest energy mode is tachyonic, and the massless Goldstone mode is a higher level excitation. The Goldstone mode for our kink solution can be deduced by looking at the first derivative of the kink solution, $\bar{\phi}$ with respect to x. It is straightforward to see from eq. (3.9) that $\bar{\phi}'$ is proportional to the ground state (3.14) of the fluctuations. As we argued above, this lowest energy state, thanks to supersymmetry, has zero energy and hence the corresponding fluctuation mode is massless.

Let us come back to the point of soliton profile in the negative x direction. The ground state wavefunction (3.14) is the lowest lying fluctuation of the soliton, which as we just argued is massless. The profile of the ground state wavefunction shares same features as that of the soliton near $x = 0$. That is, the ground state wavefunction vanishes as $x \to 0$ with all its derivatives vanishing. The wavefunction also vanishes as $x \to \infty$. If we extend the soliton solution beyond $x = 0$ along the negative x axis, the lowest lying mode will develop a node at $x = 0$, which is a contradiction. The resolution is that x is not a correct choice of coordinate. If we



(*a*) (*b*)

Figure 3: Profiles of (a) the kink solution $\bar{\phi}$ and (b) the ground state wavefunction ψ_0 of fluctuations, in the coordinate $y = \ln x$.

work in terms of the coordinate $y = \ln x$ then we resolve both these issues. The soliton profile and the ground state wave function in this coordinate are shown in figure 3.

It is instructive to do the following field redefinition [18],

$$\phi = \exp(-\chi)\,. \tag{3.16}$$

With this field redefinition, lagrangian density of Minahan and Zwiebach model becomes

$$\mathcal{L} = -\frac{1}{2}\exp(-2\chi)\,(\partial_\mu\chi\partial^\mu\chi + \chi)\,. \tag{3.17}$$

Let us consider the same field redefinition for our problem. It is straightforward to see the the lagrangian density is

$$\mathcal{L}_n = -\frac{1}{2}\exp(-2\chi)\,(\partial_\mu\chi\partial^\mu\chi + 2Aq^n\chi^n)\,. \tag{3.18}$$

It is well known that (3.17) is identical to the two derivative reduction of the boundary string field theory action. It is tempting to speculate that the lagrangian density (3.18) for our field theory models may be related to the multicritical phenomena in the boundary string field theory. It is also interesting to ask what the interpretation is of these solitons in string field theory.

4. Conclusions

In this work, we have studied a single scalar field theory model with potentials which are unbounded. We have obtained soliton solutions to the equations of motion. A single most interesting and intriguing aspect of these solutions is that they are stable, i.e., their fluctuation spectrum does not contain any tachyonic mode.

We have addressed the questions raised in the introduction, *viz,* whether the fluctuation spectrum of lump solutions of generic potentials with a parameter p, in the $p \to \infty$ limit, is equispaced with a tachyonic mode. The answers turn out to be in the negative. A simple extension of the potential studied by Minahan and Zwiebach exhibits similar behaviour, *viz.* the lump has a gaussian profile and a tachyonic instability. The modes are still equispaced and the mass-squared of the tachyonic mode is the same as that of the fluctuation of the field about the potential maximum. However, the potential governing the fluctuation spectrum of the finite p case is *not* reflectionless, albeit of a similar class of exactly solvable models. The $p \to \infty$ limit can thus be taken in a continuous manner: one does not need to restrict p to be an integer. Thus, the property of reflectionlessness of the potential is not crucial to the argument of Minahan and Zwiebach.

For various other types of potentials, which are all unbounded, we find that the gaussian lump solution is not universal. In fact, we see that the soliton solution is not even lump-like, it is instead a kink solution. We find that the small fluctuation mass spectrum for these new solitons is not equispaced. This is because the differential equation for the fluctuation modes is a Schrödinger equation with a potential which is not that of harmonic oscillator type. The tachyonic instability observed by Minahan and Zwiebach for the gaussian lump solution is also not universal. The kink solutions we obtain are stable — there is no tachyon in the fluctuation spectrum. All the potentials we have studied have one common feature, namely, a point of inflection at $\phi = 1$, where at least two derivatives of the potential $V(\phi)$ with respect to ϕ vanish. The kink solution interpolates between the local minimum at $\phi = 0$ and the point of inflection $\phi = 1$. It approaches both these points infinitely slowly and therefore field configurations beyond $\phi = 1$ are not accessible. This, we believe, is the reason for non-existence of tachyon on the kink solution.

The new field theory models, with the fluctuation spectrum governed by a supersymmetric quantum mechanical potential, are of interest in their own right. The odd n cases considered here have unbounded potentials, but the even n cases have bounded potentials. The latter belong to supersymmetric field theories [26, 27]. The soliton solution for these potentials has the same profile (eq. (3.9)). The bosonic as well as fermionic zero-modes can be calculated exactly [28]. The relation of these potentials to tachyon condensation in superstring field theory is under investigation. Models with finite p are also being analyzed. It would be interesting to see if they have some surprise in store.

Acknowledgments

We thank Ashoke Sen for asking a question which led to this investigation. We also thank him for discussion and critical reading of the manuscript. We thank Avinash Khare, Sumathi Rao and Subrata Sur for discussion. R.V. acknowledges support by CSIR, India, under grant no. 9/679(7)/2000-EMR-I.

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